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Explanatory Notes to Website for Online Coordinate Conversions and Datum Transformations

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Abbreviations

1D	1 Dimensional
CORS	Continuously Operating Reference Station
EGM2008	Earth Gravity Model 2008
GGD	Georgia Geodetic Datum
GNSS	Global Navigation Satellite System
GRS80	Geodetic Reference System 1980
IGN	Institute Geographique National
IGS	International GNSS Service
ITRF	International Terrestrial Reference Frame
ITRS	International Terrestrial Reference System
LCC	Lambert Conformal Conic
NAPR	National Agency for Public Registry
RMS	Root Mean Square
SIDA	Swedish International Development Cooperation Agency
UTM	Universal Transverse Mercator
WB	World Bank
WGS84	World Geodetic System 1984

Executive Summary

Overview. The website provides online tools for conversion of coordinates, transformation between new and old geodetic datums, generation of epoch coordinates from observations made at other times, and computation of quasigeoid undulations (height anomalies).

Purpose. The purpose of this document is to provide the formulae used for coordinate conversions and datum transformations.

Reference information for the Geodetic Reference System in Georgia

Georgia Geodetic Datum

The Georgia Geodetic Datum (GGD) is based and aligned with the International Terrestrial Reference System (ITRS). The ellipsoid associated with the datum is the Geodetic Reference System 1980 (GRS80).

The relationship between the new datum and the ITRS is realized through the ITRF2008/IGS08 coordinates for 12 CORS stations at epoch 2011.353 computed by the Institute Geographique National (IGN) in France [IGN, 2011]. Hence, all points coordinated in terms of the Georgia Geodetic Datum have coordinates defined in terms of this epoch.

The generalized motion of points in Georgia with respect to the ITRS is modeled using the global geophysical velocity NUVEL-1A [Argus and Gordon, 1991].

The new datum is complemented by a geoid model representing the difference between GPS-determined geometric heights and gravity-related heights. The website uses Georgianquasigeoid2012 developed by Lantmäteriet (Sweden) [Alfredsson and Ågren, 2012]. Georgianquasigeoid2012 is computed by first fitting the EGM2008 model to the GPS leveling height anomalies using a 1D datum transformation. The residuals are then fitted by a residual surface using the least squares collocation method. The final quasi geoid model is obtained by adding the residual surface to EGM2008. The RMS of height anomalies across Georgia is estimated at 10 cm.

Map Projection System

Cadastral mapping in Georgia uses Lambert Conformal Conic projection [Snyder, 1987] with parameters according to Table 1.

Table 1. LCC projection on Georgia Geodetic Datum.

Area of Use	Standard Parallels		Grid Origin		False Northing	False Easting
	1st	2nd	Latitude	Longitude		
Georgia	41° 40´N	42° 40´N	42° 30´N	43° 30´E	1300000.000 m	400000.000 m

The maximum scale distortion within the zone is not more than 23 cm/km. The maximum scale distortion within the zone between 41° 40´N and 42° 40´N is not more than 5 cm/km.

Transformation to Old Datums and Projection Systems

Two geodetic datums have been used in Georgia in Soviet time and after declaring independence:

- The datum in Soviet time consisted of two separate classical datums: a horizontal datum (Pulkovo 1942) which formed the basis for the computations of horizontal control surveys, and a vertical datum (Baltic Height System 1977) to which elevations are referred. The map projection was Gauss-Krüger. Georgia was situated in zones 7 and 8.

- The WGS84 datum was introduced in 1999 through the coordinates of 5 stations computed at the Massachusetts Institute of Technology in the frame of their geodynamic project for studying plate kinematics and dynamics in the eastern Mediterranean and Caucasus. The map projection was Universal Transverse Mercator. Georgia is situated in zones 37 and 38.

Transformation Parameters between GGD and WGS84

The transformation parameters between GGD and WGS84 are computed from 5 identical points using the Bursa-Wolf model (Table 2). This transformation can be applied countrywide.

Table 2. Parameters of Bursa-Wolf transformation between GGD and WGS84.

Bursa-Wolf:

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = (1+s)\begin{bmatrix} 1 & -q & b \\ q & 1 & -a \\ -b & a & 1 \end{bmatrix}\begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}.$$

	From GGD to WGS84	**From WGS84 to GGD**
ΔX [m]	-2.0796 ±1.4858	2.0796 ±1.4858
ΔY [m]	-0.3484 ±1.4431	0.3484 ±1.4431
ΔZ [m]	1.7009 ±2.0078	-1.7009 ±2.0078
a ["]	0.05465 ±0.06158	-0.05465 ±0.06158
b ["]	-0.06718 ±0.06362	0.06718 ±0.06362
q ["]	0.06143 ±0.02982	-0.06143 ±0.02982
s [ppm]	0.0181 ±0.1393	-0.0181 ±0.1393
Points	5	5
Post Fit [m]	0.0340	0.0340

Transformation Parameters between GGD and Pulkovo 1942

The transformation parameters between GGD and Pulkovo 1942 are computed from 16 identical points using the Bursa-Wolf model (Table 3). This transformation should be applied on the surface of the ellipsoid and is valid countrywide.

Table 3. Parameters of Bursa-Wolf transformation between GGD and Pulkovo 1942.

Bursa-Wolf:

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \end{bmatrix} = (1+s)\begin{bmatrix} 1 & -q & b \\ q & 1 & -a \\ -b & a & 1 \end{bmatrix}\begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}.$$

	From GGD to Pulkovo 1942	From Pulkovo 1942 to GGD
ΔX [m]	40.7436 ±5.4889	-40.7436 ±5.4889
ΔY [m]	40.0018 ±2.0974	-40.0018 ±2.0974
ΔZ [m]	56.7070 ±5.6937	-56.7070 ±5.6937
a ["]	1.27530 ±0.13360	-1.27530 ±0.13360
b ["]	1.42112 ±0.21582	-1.42112 ±0.21582
q ["]	-2.69445 ±0.07631	2.69445 ±0.07631
s [ppm]	4.5284 ±0.2185	-4.5284 ±0.2185
Points	16	16
Post Fit [m]	0.1089	0.1089

References

Alfredsson, A. and J. Ågren (2012): Report on the calculation of the Georgian quasi geoid model 2012, SIDA/WB project Capacity Building and Improved Client Services at NAPR of Georgia, 2007-002228, Report GEL1 F-08, November, 2012.

Argus, D.F. and R.G. Gordon (1991): No-Net-Rotation Model of Current Plate Velocities Incorporating Plate Motion Model NUVEL-1, *Geophys. Res. Lett.* 18 (11), 2039–2042.

IGN (2011): Georgia Continuously Operating Reference Stations Coordinates Computation Report Version 1.0, Institut Géographique National, May 25, 2011.

Snyder, J.P. (1987): Map Projections: A Working Manual, USGS Professional Paper: 1395, 99, USGS.

Coordinate Conversions

Cartesian Coordinates from Geographic Coordinates on Georgia Geodetic Datum

Input	SN, X, Y, Z
Conversion from geographic coordinates to Cartesian coordinates	$d = \left(X^2 + Y^2\right)^{½}$ $\varphi_{ini} = arctan\left[Z\left(1 + e'^2\right) / d\right]$ $\varphi_{i+1} = arctan\left\{\left[Z + N_e e^2 sin\varphi_i\right] / d\right\}$ (is calculated iterative; loop continued to iterate until the difference in φ became less than 0.00005 and the difference in h became less than 0.0005) $\lambda = arctan\left(Y / X\right)$ $h = d / cos\varphi - N_e$ $a = 6378137$ $f = 1/298.257222101$ $b = a\left(1 - f\right)$ $e = \left[\left(a^2 - b^2\right) / a^2\right]^{½}$ $e' = \left[\left(a^2 - b^2\right) / b^2\right]^{½}$ $N_e = a / \left[1 - e^2 sin^2\varphi\right]^{½}$
Output	SN, φ, λ, h

Geographic Coordinates on Georgia Geodetic Datum from Cartesian Coordinates

Input	SN, φ, λ, h
Conversion from Cartesian coordinates to geographic coordinates	$X = (N_e + h)\cos\varphi\cos\lambda$ $Y = (N_e + h)\cos\varphi\sin\lambda$ $Z = [N_e(1 - e^2) + h)\sin\varphi$ $a = 6378137$ $f = 1/298.257222101$ $b = a(1 - f)$ $e = [(a^2 - b^2) / a^2]^{½}$ $N_e = a / [1 - e^2\sin^2\varphi]^{½}$
Output	SN, X, Y, Z

Lambert Coordinates from Latitude and Longitude on Georgia Geodetic Datum

Input	SN, φ, λ
Computation of Lambert coordinates from latitude and longitude	$\varphi_1 = 41° \ 40'$ $\varphi_2 = 42° \ 40'$ $\varphi_0 = 42° \ 30'$ $\lambda_0 = 43° \ 30'$ $a = 6378137$ $f = 1/298.257222101$ $b = a(1 - f)$ $e = \left[(a^2 - b^2) \ / \ a^2\right]^{½}$ $m_1 = \cos\varphi_1 / (1 - e^2 \sin^2\varphi_1)^{½}$ $m_2 = \cos\varphi_2 / (1 - e^2 \sin^2\varphi_2)^{½}$ $t = \left[\left(\frac{1 - \sin\varphi}{1 + \sin\varphi}\right)\left(\frac{1 + e\sin\varphi}{1 - e\sin\varphi}\right)^e\right]^{½}$ $t_0 = \left[\left(\frac{1 - \sin\varphi_0}{1 + \sin\varphi_0}\right)\left(\frac{1 + e\sin\varphi_0}{1 - e\sin\varphi_0}\right)^e\right]^{½}$ $t_1 = \left[\left(\frac{1 - \sin\varphi_1}{1 + \sin\varphi_1}\right)\left(\frac{1 + e\sin\varphi_1}{1 - e\sin\varphi_1}\right)^e\right]^{½}$ $t_2 = \left[\left(\frac{1 - \sin\varphi_2}{1 + \sin\varphi_2}\right)\left(\frac{1 + e\sin\varphi_2}{1 - e\sin\varphi_2}\right)^e\right]^{½}$ $n = (\ln m_1 - \ln m_2) / (\ln t_1 - \ln t_2)$ $F = m_1 / (n t_1^n)$ $\rho_0 = aFt_0^n$ $\rho = aFt^n$ $\theta = n(\lambda - \lambda_0)$ $x = \rho\sin\theta$ $y = \rho_0 - \rho\cos\theta$ $N^L = 1300000.0000 + y$ $E^L = 400000.0000 + x$
Output	SN, N_L, E_L

Latitude and Longitude from Lambert Coordinates on Georgia Geodetic Datum

Input	SN, N^L, E^L
Computation of latitude and longitude from Lambert coordinates for a single projection zone	$\varphi_1 = 41° \; 40'$ $\varphi_2 = 42° \; 40'$ $\varphi_0 = 42° \; 30'$ $\lambda_0 = 43° \; 30'$ $a = 6378137$ $f = 1/298.257222101$ $b = a(1 - f)$ $e = \left[(a^2 - b^2) \; / \; a^2\right]^{1/2}$ $y = N^L - 1300000.0000$ $x = E^L - 400000.0000$ $m_1 = \cos\varphi_1 / \left(1 - e^2 \sin^2\varphi_1\right)^{1/2}$ $m_2 = \cos\varphi_2 / \left(1 - e^2 \sin^2\varphi_2\right)^{1/2}$ $t_0 = \left[\left(\frac{1-\sin\varphi_0}{1+\sin\varphi_0}\right)\left(\frac{1+e\sin\varphi_0}{1-e\sin\varphi_0}\right)^e\right]^{1/2}$ $t_1 = \left[\left(\frac{1-\sin\varphi_1}{1+\sin\varphi_1}\right)\left(\frac{1+e\sin\varphi_1}{1-e\sin\varphi_1}\right)^e\right]^{1/2}$ $t_2 = \left[\left(\frac{1-\sin\varphi_2}{1+\sin\varphi_2}\right)\left(\frac{1+e\sin\varphi_2}{1-e\sin\varphi_2}\right)^e\right]^{1/2}$ $n = (\ln m_1 - \ln m_2) / (\ln t_1 - \ln t_2)$ $F = m_1 / (n t_1^n)$ $\rho_0 = aFt_0^n$ $\rho = \pm\left[x^2 + (\rho_0 - y)^2\right]^{1/2}$, taking the sign of n $t = (\rho / aF)^{1/n}$ $\theta = \arctan\left[x / (\rho_0 - y)\right]$ $\eta = \pi/2 - 2\arctan t$ $\varphi = \eta + \left(e^2/2 + 5e^4/24 + e^6/12 + 13e^8/360 + \ldots\right)\sin 2\eta +$ $+\left(7e^4/48 + 29e^6/240 + 811e^8/11520 + \ldots\right)\sin 4\eta +$ $+\left(7e^6/120 + 81e^8/1120 + \ldots\right)\sin 6\eta +$ $+\left(4279e^8/161280 + \ldots\right)\sin 8\eta + \ldots$ $\lambda = \theta / n + \lambda_0$
Output	SN, φ, λ

UTM Coordinates for Zone 37 from Latitude and Longitude on Georgia Geodetic Datum

Input	SN, φ, λ
Computation of UTM coordinates for zone 37 from latitude and longitude	$k_o = 0.9996$ $\varphi_0 = 0.0000$ $\lambda_0 = 39 * 180 / 3.14159265358979$ $a = 6378137$ $f = 1/298.257222101$ $b = a(1 - f)$ $e = \left[(a^2 - b^2) / a^2\right]^{½}$ $e'^2 = e^2/(1-e^2)$ $N = a/(1-e^2\sin^2\varphi)^{1/2}$ $T = \tan^2\varphi$ $C = e'^2\cos^2\varphi$ $A = (\lambda - \lambda_0)\cos\varphi$, with λ and λ_0 in radians. $M = a[(1- e^2/4 - 3e^4/64 - 5e^6/256 - ...)\varphi -$ $(3e^2/8 + 3e^4/32 + 45e^6/1024 + ...)\sin 2\varphi +$ $(15e^4/256 + 45e^6/1024 + ...)\sin 4\varphi -$ $(35e^6/3072 + ...)\sin 6\varphi + ...]$ with φ in radians. $M_0 = a[(1- e^2/4 - 3e^4/64 - 5e^6/256 - ...)\varphi_0 -$ $(3e^2/8 + 3e^4/32 + 45e^6/1024 + ...)\sin 2\varphi_0 +$ $(15e^4/256 + 45e^6/1024 + ...)\sin 4\varphi_0 -$ $(35e^6/3072 + ...)\sin 6\varphi_0 + ...]$ with φ_0 in radians. $x = k_0N\left[A + (1-T+C)A^3/6 + (5-18T+T^2+72C-58e'^2)A^5/120\right]$ $y = k_0\{M - M_0 + N\tan\varphi[A^2/2 + (5-T+9C+4C^2)A^4/24 +$ $(61-58T+T^2+600C-330e'^2)A^6/720]\}$ $N_{37}^{UTM} = y$ $E_{37}^{UTM} = 500000.0000 + x$
Output	$SN, N_{37}^{UTM}, E_{37}^{UTM}$

Latitude and Longitude from UTM Coordinates for Zone 37 on Georgia Geodetic Datum

Input	$SN, N_{37}^{UTM}, E_{37}^{UTM}$
Computation of latitude and longitude from of UTM coordinates for zone 37	$k_o = 0.9996$ $\varphi_0 = 0.0000$ $\lambda_0 = 39 * 3.14159265358979 / 180$ $a = 6378137$ $f = 1/298.257222101$ $b = a(1 - f)$ $e = \left[\left(a^2 - b^2\right) / a^2\right]^{½}$ $y = N_{37}^{UTM}$ $x = E_{37}^{UTM} - 500000.0000$ $M_0 = a\left[\left(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...\right)\varphi_0 -\right.$ $\left(3e^2/8 + 3e^4/32 + 45e^6/1024 + ...\right)\sin 2\varphi_0 +$ $\left.\left(15e^4/256 + 45e^6/1024 + ...\right)\sin 4\varphi_0 - \left(35e^6/3072 + ...\right)\sin 6\varphi_0 + ...\right]$ with φ_0 in radians. $M = M_0 + y/k_0$ $\mu = M/\left[a\left(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...\right)\right]$ $e_1 = \left[1 - \left(1 - e^2\right)^{1/2}\right] / \left[1 + \left(1 - e^2\right)^{1/2}\right]$ $\varphi_1 = \mu + \left(3e_1/2 - 27e_1^3/32 + ...\right)\sin 2\mu + \left(21e_1^2/16 - 55e_1^4/32 + ...\right)\sin 4\mu$ $+ \left(151e_1^3/96 + ...\right)\sin 6\mu + \left(1097e_1^4/512 + ...\right)\sin 8\mu + ...$ $e'^2 = e^2/(1 - e^2)$ $C_1 = e'^2 \cos^2 \varphi_1$ $T_1 = \tan^2 \varphi_1$ $N_1 = a/\left(1 - e^2 \sin^2 \varphi_1\right)^{1/2}$ $R_1 = a\left(1 - e^2\right) / \left(1 - e^2 \sin^2 \varphi_1\right)^{3/2}$ $D = x/\left(N_1 k_0\right)$ $\varphi = \varphi_1 - \left(N_1 \tan\varphi_1 / R_1\right)\left[D^2/2 - \left(5 + 3T_1 + 10C_1 - 4C_1^2 - 9e'^2\right)D^4/24 +\right.$ $\left.\left(61 + 90T_1 + 298C_1 + 45T_1^2 - 252e'^2 - 3C_1^2\right)D^6/720\right]$ $\lambda = \lambda_0 + \left[D - \left(1 + 2T_1 + C_1\right)D^3/6 +\right.$ $\left.\left(5 - 2C_1 + 28T_1 - 3C_1^2 + 8e'^2 + 24T_1^2\right)D^5/120\right]/\cos\varphi_1$
Output	SN, φ, λ

UTM Coordinates for Zone 38 from Latitude and Longitude on Georgia Geodetic Datum

Input	SN, φ, λ
Computation of UTM coordinates for zone 38 from latitude and longitude	$k_o = 0.9996$ $\varphi_0 = 0.0000$ $\lambda_0 = 45 * 3.14159265358979 / 180$ $a = 6378137$ $f = 1/298.257222101$ $b = a(1 - f)$ $e = \left[\left(a^2 - b^2\right) / a^2\right]^{½}$ $e'^2 = e^2/(1-e^2)$ $N = a/\left(1-e^2\sin^2\varphi\right)^{1/2}$ $T = \tan^2\varphi$ $C = e'^2\cos^2\varphi$ $A = (\lambda - \lambda_0)\cos\varphi$, with λ and λ_0 in radians. $M = a\left[\left(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...\right)\varphi - \left(3e^2/8 + 3e^4/32 + 45e^6/1024 + ...\right)\sin 2\varphi + \left(15e^4/256 + 45e^6/1024 + ...\right)\sin 4\varphi - \left(35e^6/3072 + ...\right)\sin 6\varphi + ...\right]$ with φ in radians. $M_0 = a\left[\left(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...\right)\varphi_0 - \left(3e^2/8 + 3e^4/32 + 45e^6/1024 + ...\right)\sin 2\varphi_0 + \left(15e^4/256 + 45e^6/1024 + ...\right)\sin 4\varphi_0 - \left(35e^6/3072 + ...\right)\sin 6\varphi_0 + ...\right]$ with φ_0 in radians. $x = k_0 N\left[A + (1 - T + C)A^3/6 + \left(5 - 18T + T^2 + 72C - 58e'^2\right)A^5/120\right]$ $y = k_0\left\{M - M_0 + N\tan\varphi\left[A^2/2 + \left(5 - T + 9C + 4C^2\right)A^4/24 + \left(61 - 58T + T^2 + 600C - 330e'^2\right)A^6/720\right]\right\}$ $N_{38}^{UTM} = y$ $E_{38}^{UTM} = 500000.0000 + x$
Output	$SN, N_{38}^{UTM}, E_{38}^{UTM}$

Latitude and Longitude from UTM Coordinates for Zone 38 on Georgia Geodetic Datum

Input	$SN, N_{38}^{UTM}, E_{38}^{UTM}$
Computation of latitude and longitude from of UTM coordinates for zone 38	$k_o = 0.9996$ $\varphi_0 = 0.0000$ $\lambda_0 = 45 * 3.14159265358979 / 180$ $a = 6378137$ $f = 1/298.257222101$ $b = a(1 - f)$ $e = \left[(a^2 - b^2) / a^2\right]^{1/2}$ $y = N_{38}^{UTM}$ $x = E_{38}^{UTM} - 500000.0000$ $M_0 = a\left[(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...)\varphi_0 - (3e^2/8 + 3e^4/32 + 45e^6/1024 + ...)\sin 2\varphi_0 + (15e^4/256 + 45e^6/1024 + ...)\sin 4\varphi_0 - (35e^6/3072 + ...)\sin 6\varphi_0 + ...\right]$ with φ_0 in radians. $M = M_0 + y/k_0$ $\mu = M/\left[a(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...)\right]$ $e_1 = \left[1 - (1 - e^2)^{1/2}\right] / \left[1 + (1 - e^2)^{1/2}\right]$ $\varphi_1 = \mu + (3e_1/2 - 27e_1^3/32 + ...)\sin 2\mu + (21e_1^2/16 - 55e_1^4/32 + ...)\sin 4\mu + (151e_1^3/96 + ...)\sin 6\mu + (1097e_1^4/512 + ...)\sin 8\mu + ...$ $e'^2 = e^2/(1 - e^2)$ $C_1 = e'^2 \cos^2 \varphi_1$ $T_1 = \tan^2 \varphi_1$ $N_1 = a/(1 - e^2 \sin^2 \varphi_1)^{1/2}$ $R_1 = a(1 - e^2) / (1 - e^2 \sin^2 \varphi_1)^{3/2}$ $D = x/(N_1 k_0)$ $\varphi = \varphi_1 - (N_1 \tan\varphi_1 / R_1)\left[D^2/2 - (5 + 3T_1 + 10C_1 - 4C_1^2 - 9e'^2)D^4/24 + (61 + 90T_1 + 298C_1 + 45T_1^2 - 252e'^2 - 3C_1^2)D^6/720\right]$ $\lambda = \lambda_0 + \left[D - (1 + 2T_1 + C_1)D^3/6 + (5 - 2C_1 + 28T_1 - 3C_1^2 + 8e'^2 + 24T_1^2)D^5/120\right]/\cos\varphi_1$
Output	SN, φ, λ

Gauss-Krüger Coordinates for Zone 7 from Latitude and Longitude on Pulkovo 1942 Datum

Input	SN, φ, λ
Computation of Gauss-Krüger coordinates for Zone 7 from latitude and longitude	$k_o = 1.0000$ $\varphi_0 = 0.0000$ $\lambda_0 = 39 * 3.14159265358979 / 180$ $a = 6378245$ $f = 1/298.3$ $b = a(1 - f)$ $e = \left[(a^2 - b^2) / a^2\right]^{1/2}$ $e'^2 = e^2/(1-e^2)$ $N = a/(1-e^2\sin^2\varphi)^{1/2}$ $T = \tan^2\varphi$ $C = e'^2\cos^2\varphi$ $A = (\lambda - \lambda_0)\cos\varphi$, with λ and λ_0 in radians. $M = a\left[(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...)\varphi - \right.$ $(3e^2/8 + 3e^4/32 + 45e^6/1024 + ...)\sin 2\varphi +$ $(15e^4/256 + 45e^6/1024 + ...)\sin 4\varphi -$ $\left.(35e^6/3072 + ...)\sin 6\varphi + ...\right]$ with φ in radians. $M_0 = a\left[(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...)\varphi_0 - \right.$ $(3e^2/8 + 3e^4/32 + 45e^6/1024 + ...)\sin 2\varphi_0 +$ $(15e^4/256 + 45e^6/1024 + ...)\sin 4\varphi_0 -$ $\left.(35e^6/3072 + ...)\sin 6\varphi_0 + ...\right]$ with φ_0 in radians. $x = k_0 N\left[A + (1 - T + C)A^3/6 + (5 - 18T + T^2 + 72C - 58e'^2)A^5/120\right]$ $y = k_0\left\{M - M_0 + N\tan\varphi\left[A^2/2 + (5 - T + 9C + 4C^2)A^4/24 + \right.\right.$ $\left.\left.(61 - 58T + T^2 + 600C - 330e'^2)A^6/720\right]\right\}$ $N_7^{1942} = y$ $E_7^{1942} = 7500000.0000 + x$
Output	$SN, N_7^{1942}, E_7^{1942}$

Latitude and Longitude from Gauss-Krüger Coordinates for Zone 7 on Pulkovo 1942 Datum

Input	$SN, N_7^{1942}, E_7^{1942}$
Computation of latitude and longitude from of Gauss-Krüger Coordinates for Zone 7	$k_o = 1.0000$ $\varphi_0 = 0.0000$ $\lambda_0 = 39 * 3.14159265358979 / 180$ $a = 6378245$ $f = 1/298.3$ $b = a(1 - f)$ $e = \left[(a^2 - b^2) / a^2\right]^{½}$ $y = N_7^{1942}$ $x = E_7^{1942} - 7500000.0000$ $M_0 = a\left[(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...)\varphi_0 - \right.$ $(3e^2/8 + 3e^4/32 + 45e^6/1024 + ...)\sin 2\varphi_0 +$ $\left.(15e^4/256 + 45e^6/1024 + ...)\sin 4\varphi_0 - (35e^6/3072 + ...)\sin 6\varphi_0 + ...\right]$ with φ_0 in radians. $M = M_0 + y/k_0$ $\mu = M/\left[a(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...)\right]$ $e_1 = \left[1 - (1 - e^2)^{1/2}\right]/\left[1 + (1 - e^2)^{1/2}\right]$ $\varphi_1 = \mu + (3e_1/2 - 27e_1^3/32 + ...)\sin 2\mu + (21e_1^2/16 - 55e_1^4/32 + ...)\sin 4\mu$ $+ (151e_1^3/96 + ...)\sin 6\mu + (1097e_1^4/512 + ...)\sin 8\mu + ...$ $e'^2 = e^2/(1 - e^2)$ $C_1 = e'^2 \cos^2 \varphi_1$ $T_1 = \tan^2 \varphi_1$ $N_1 = a/(1 - e^2 \sin^2 \varphi_1)^{1/2}$ $R_1 = a(1 - e^2) / (1 - e^2 \sin^2 \varphi_1)^{3/2}$ $D = x/(N_1 k_0)$ $\varphi = \varphi_1 - (N_1 \tan\varphi_1 / R_1)\left[D^2/2 - (5 + 3T_1 + 10C_1 - 4C_1^2 - 9e'^2)D^4/24 + \right.$ $\left.(61 + 90T_1 + 298C_1 + 45T_1^2 - 252e'^2 - 3C_1^2)D^6/720\right]$ $\lambda = \lambda_0 + \left[D - (1 + 2T_1 + C_1)D^3 / 6 + \right.$ $\left.(5 - 2C_1 + 28T_1 - 3C_1^2 + 8e'^2 + 24T_1^2)D^5/120\right]/\cos\varphi_1$
Output	SN, φ, λ

Gauss-Krüger Coordinates for Zone 8 from Latitude and Longitude on Pulkovo 1942 Datum

Input	SN, φ, λ
Computation of Gauss-Krüger coordinates for Zone 8 from latitude and longitude	$k_o = 1.0000$ $\varphi_0 = 0.0000$ $\lambda_0 = 45 * 3.14159265358979 / 180$ $a = 6378245$ $f = 1/298.3$ $b = a(1 - f)$ $e = \left[\left(a^2 - b^2\right) / a^2\right]^{½}$ $e'^2 = e^2/(1-e^2)$ $N = a/\left(1-e^2\sin^2\varphi\right)^{1/2}$ $T = \tan^2\varphi$ $C = e'^2\cos^2\varphi$ $A = (\lambda - \lambda_0)\cos\varphi$ with, λ and λ_0 in radians. $\mathrm{M} = \mathrm{a}\left[\left(1 - \mathrm{e}^2/4 - 3\mathrm{e}^4/64 - 5\mathrm{e}^6/256 - ...\right)\varphi - \right.$ $\left(3e^2/8 + 3e^4/32 + 45e^6/1024 + ...\right)\sin 2\varphi +$ $\left(15e^4/256 + 45e^6/1024 + ...\right)\sin 4\varphi -$ $\left.\left(35e^6/3072 + ...\right)\sin 6\varphi + ...\right]$ with φ in radians. $\mathrm{M}_0 = \mathrm{a}\left[\left(1 - \mathrm{e}^2/4 - 3\mathrm{e}^4/64 - 5\mathrm{e}^6/256 - ...\right)\varphi_0 - \right.$ $\left(3e^2/8 + 3e^4/32 + 45e^6/1024 + ...\right)\sin 2\varphi_0 +$ $\left(15e^4/256 + 45e^6/1024 + ...\right)\sin 4\varphi_0 -$ $\left.\left(35e^6/3072 + ...\right)\sin 6\varphi_0 + ...\right]$ with φ_0 in radians. $x = k_0 N\left[A + (1 - T + C)A^3/6 + \left(5 - 18T + T^2 + 72C - 58e'^2\right)A^5/120\right]$ $y = k_0\left\{M - M_0 + N\tan\varphi\left[A^2/2 + \left(5 - T + 9C + 4C^2\right)A^4/24 + \right.\right.$ $\left.\left.\left(61 - 58T + T^2 + 600C - 330e'^2\right)A^6/720\right]\right\}$ $N_8^{1942} = y$ $E_8^{1942} = 8500000.0000 + x$
Output	$SN, N_8^{1942}, E_8^{1942}$

Latitude and Longitude from Gauss-Krüger Coordinates for Zone 8 on Pulkovo 1942 Datum

Input	$SN, N_8^{1942}, E_8^{1942}$
Computation of latitude and longitude from of Gauss-Krüger Coordinates for Zone 8	$k_o = 1.0000$ $\varphi_0 = 0.0000$ $\lambda_0 = 45 * 3.14159265358979 / 180$ $a = 6378245$ $f = 1/298.3$ $f = 298.3$ $b = a(1 - f)$ $e = \left[(a^2 - b^2) / a^2\right]^{½}$ $y = N_8^{1942}$ $x = E_8^{1942} - 8500000.0000$ $M_0 = a\left[\left(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...\right)\varphi_0 - \right.$ $\left(3e^2/8 + 3e^4/32 + 45e^6/1024 + ...\right)\sin 2\varphi_0 +$ $\left.\left(15e^4/256 + 45e^6/1024 + ...\right)\sin 4\varphi_0 - \left(35e^6/3072 + ...\right)\sin 6\varphi_0 + ...\right]$ with φ_0 in radians. $M = M_0 + y/k_0$ $\mu = M/\left[a\left(1 - e^2/4 - 3e^4/64 - 5e^6/256 - ...\right)\right]$ $e_1 = \left[1 - \left(1 - e^2\right)^{1/2}\right] / \left[1 + \left(1 - e^2\right)^{1/2}\right]$ $\varphi_1 = \mu + \left(3e_1/2 - 27e_1^3/32 + ...\right)\sin 2\mu + \left(21e_1^2/16 - 55e_1^4/32 + ...\right)\sin 4\mu$ $+ \left(151e_1^3/96 + ...\right)\sin 6\mu + \left(1097e_1^4/512 + ...\right)\sin 8\mu + ...$ $e'^2 = e^2/(1 - e^2)$ $C_1 = e'^2 \cos^2 \varphi_1$ $T_1 = \tan^2 \varphi_1$ $N_1 = a/\left(1 - e^2 \sin^2 \varphi_1\right)^{1/2}$ $R_1 = a\left(1 - e^2\right) / \left(1 - e^2 \sin^2 \varphi_1\right)^{3/2}$ $D = x/(N_1 k_0)$ $\varphi = \varphi_1 - \left(N_1 \tan\varphi_1 / R_1\right)\left[D^2/2 - \left(5 + 3T_1 + 10C_1 - 4C_1^2 - 9e'^2\right)D^4/24 + \right.$ $\left.\left(61 + 90T_1 + 298C_1 + 45T_1^2 - 252e'^2 - 3C_1^2\right)D^6/720\right]$ $\lambda = \lambda_0 + \left[D - \left(1 + 2T_1 + C_1\right)D^3 / 6 + \right.$ $\left.\left(5 - 2C_1 + 28T_1 - 3C_1^2 + 8e'^2 + 24T_1^2\right)D^5/120\right]/\cos\varphi_1$
Output	SN, φ, λ

Datum Transformations

Cartesian Coordinates from Georgia Geodetic Datum to WGS84 Datum

Input	$SN, X_{GGD}, Y_{GGD}, Z_{GGD}$
Conversion of Cartesian coordinates from Georgia Geodetic Datum to WGS84	$\begin{bmatrix} X_{WGS84} \\ Y_{WGS84} \\ Z_{WGS84} \end{bmatrix} = \left(1 + k/10^6\right) \begin{bmatrix} 1 & R_z & -R_y \\ -R_z & 1 & R_x \\ R_y & -R_x & 1 \end{bmatrix} \begin{bmatrix} X_{GGD} \\ Y_{GGD} \\ Z_{GGD} \end{bmatrix} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$ $\Delta X = -2.0796$ m $\Delta Y = -0.3484$ m $\Delta Z = +1.7009$ m $R_x = +0.05465''$ $R_y = -0.06718''$ $R_z = +0.06143''$ $k = +0.0181$
Output	$SN, X_{WGS84}, Y_{WGS84}, Z_{WGS84}$

Cartesian Coordinates from WGS84 Datum to Georgia Geodetic Datum

Input	$SN, X_{WGS84}, Y_{WGS84}, Z_{WGS84}$
Conversion of Cartesian coordinates from WGS84 to Georgia Geodetic Datum	$\begin{bmatrix} X_{GGD} \\ Y_{GGD} \\ Z_{GGD} \end{bmatrix} = \left(1 + k/10^6\right) \begin{bmatrix} 1 & R_z & -R_y \\ -R_z & 1 & R_x \\ R_y & -R_x & 1 \end{bmatrix} \begin{bmatrix} X_{WGS84} \\ Y_{WGS84} \\ Z_{WGS84} \end{bmatrix} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$ $\Delta X = +2.0796$ m $\Delta Y = +0.3484$ m $\Delta Z = -1.7009$ m $R_x = -0.05465''$ $R_y = +0.06718''$ $R_z = -0.06143''$ $k = -0.0181$
Output	$SN, X_{GGD}, Y_{GGD}, Z_{GGD}$

Cartesian coordinates from Georgia Geodetic Datum to Pulkovo 1942 Datum

Input	$SN, X_{GGD}, Y_{GGD}, Z_{GGD}$
Conversion of Cartesian coordinates from Georgia Geodetic Datum to Pulkovo 1942	$\begin{bmatrix} X_{1942} \\ Y_{1942} \\ Z_{1942} \end{bmatrix} = \left(1 + k/10^6\right) \begin{bmatrix} 1 & R_z & -R_y \\ -R_z & 1 & R_x \\ R_y & -R_x & 1 \end{bmatrix} \begin{bmatrix} X_{GGD} \\ Y_{GGD} \\ Z_{GGD} \end{bmatrix} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$ $\Delta X = +40.7436\text{ m}$ $\Delta Y = +40.0018\text{ m}$ $\Delta Z = +56.7070\text{ m}$ $R_x = +1.27530''$ $R_y = +1.42112''$ $R_z = -2.69445''$ $k = +4.5284$
Output	$SN, X_{1942}, Y_{1942}, Z_{1942}$

Cartesian Coordinates from Pulkovo 1942 Datum to Georgia Geodetic Datum

Input	$SN, X_{1942}, Y_{1942}, Z_{1942}$
Conversion of Cartesian coordinates from Pulkovo 1942 to Georgia Geodetic Datum	$\begin{bmatrix} X_{GGD} \\ Y_{GGD} \\ Z_{GGD} \end{bmatrix} = \left(1 + k/10^6\right) \begin{bmatrix} 1 & R_z & -R_y \\ -R_z & 1 & R_x \\ R_y & -R_x & 1 \end{bmatrix} \begin{bmatrix} X_{1942} \\ Y_{1942} \\ Z_{1942} \end{bmatrix} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$ $\Delta X = -40.7436$ m $\Delta Y = -40.0018$ m $\Delta Z = -56.7070$ m $R_x = -1.27530''$ $R_y = -1.42112''$ $R_z = +2.69445''$ $k = -4.5284$
Output	$SN, X_{GGD}, Y_{GGD}, Z_{GGD}$

Epoch Coordinates on Georgia Geodetic Datum from Observations Made at Other Times

Input	SN, X, Y, Z, t_0, t
Generation of epoch coordinates from observations made at other times.	$X_t = X + \Delta t V_X / 1000$ $Y_t = Y + \Delta t V_Y / 1000$ $Z_t = Z + \Delta t V_Z / 1000$ $V_X = -E_{VEL} \sin\lambda - N_{VEL} \sin\varphi \cos\varphi$ $V_Y = E_{VEL} \cos\lambda - N_{VEL} \sin\varphi \sin\varphi$ $V_X = N_{VEL} \cos\varphi$ E_{VEL}, N_{VEL} are computed from the geophysical model NUVEL 1A by entering X, Y, Z and the year fraction Δt of the number of days between t and t_0.
Ouput	SN, X_t, Y_t, Z_t

Quasigeoid

Height Anomaly from Geographic Coordinates on Georgia Geodetic Datum

Input	SN, φ, λ, h
Computation of height anomaly	The height anomaly ($N_{quasigeoid}$) at the point with coordinates φ, λ is interpolated from the Georgianquasigeoid2012 $H = h - N_{quasigeoid}$
Ouput	$SN, \varphi, \lambda, H, N_{quasigeoid}$